



Lecture 5

Reading Material



Reading:

Chapter 6 Solymar and Walsh

Fermi-Dirac Statistics

Region I: $E_F - E \gg k_B T$

$$F(E) \approx 1 - \exp\left[-\frac{E - E_F}{k_B T}\right] \approx 1$$

Approximately Unity

Region II: $E_F - E \gg k_B T$

$$0 \leq F(E) \leq 1$$

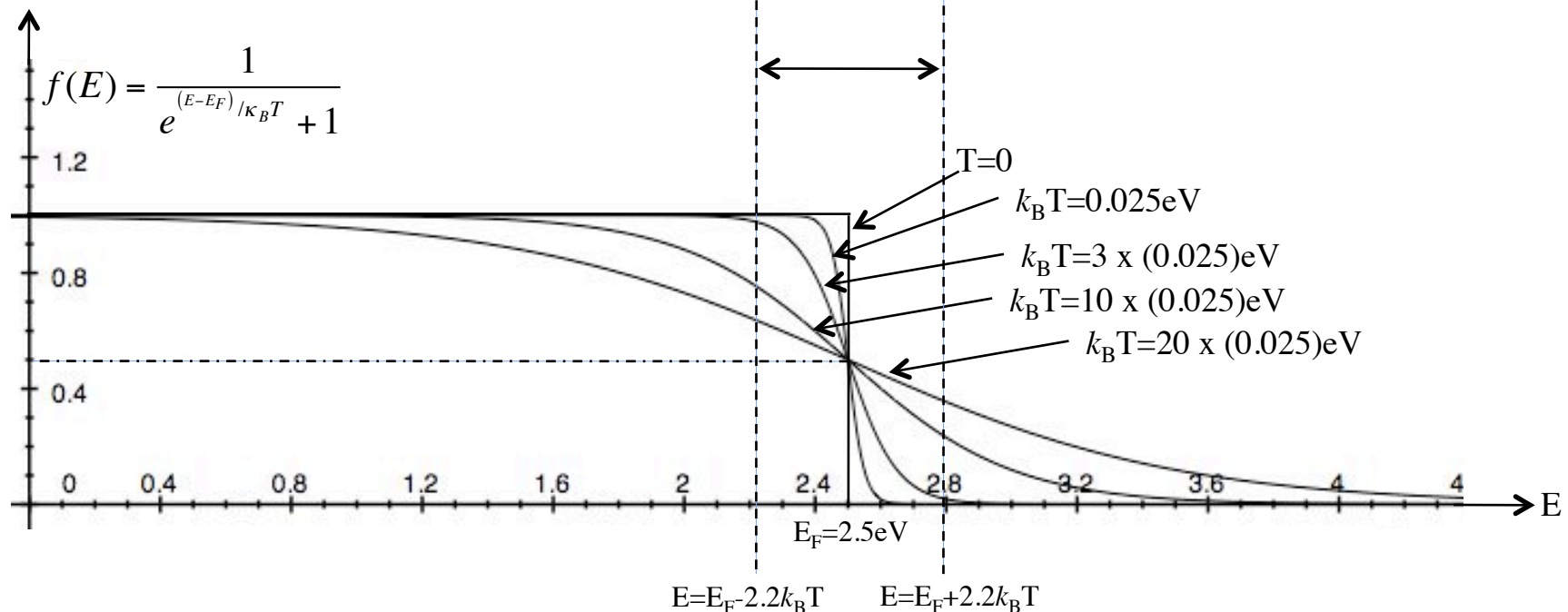
Slope is function of T

10% - 90% Width $\approx 4.4 k_B T$

Region III: $E_F - E \gg k_B T$

$$F(E) \approx \exp\left[-\frac{E - E_F}{k_B T}\right]$$

Maxwell - Boltzman distribution tail



Specific Heat of Electrons



- Classical theory does not predict the specific heat of electrons
- Due to Pauli Exclusion Principle, the lower energy electrons cannot be heated up into the next energy state that are occupied by electrons.
- Therefore it is the electrons near or at the Fermi level that can be heated up and contribute to the specific heat because there are available energy states to go into
- The specific heat is defined as

$$c_V = \frac{d\langle E \rangle}{dT}$$

Specific Heat of Electrons

- Assume that all electrons with energy

$$E_F - 2.2k_B T \leq E \leq E_F$$

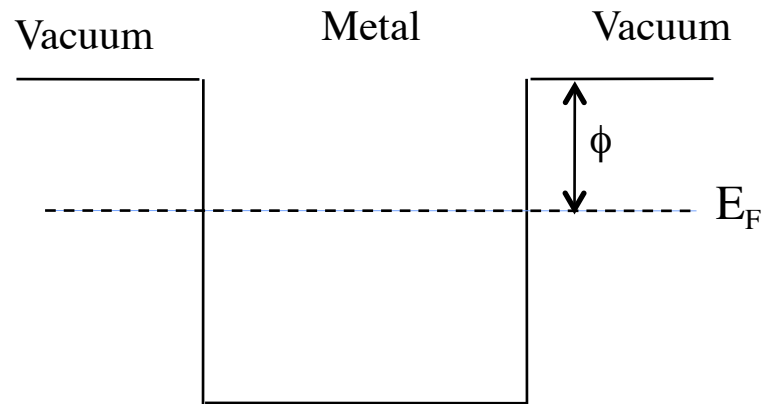
contribute to the specific heat and can be treated as classical electrons (i.e. there are plenty of available states to excite them into)

- Assume each classical electron has an average energy $3/2k_B T$, then the average energy of the electrons that can contribute to the SH and the resulting specific heat of these electrons is given by

$$\langle E \rangle = \frac{3}{2} k_B T \frac{2.2 k_B T}{E_F}$$
$$c_V = \frac{d\langle E \rangle}{dT} = \frac{d}{dT} \left[\frac{3}{2} k_B T \frac{2.2 k_B T}{E_F} \right] = 6.6 k_B^2 \frac{T}{E_F}$$

The Work Function

- The threshold energy required to remove an electron from the metal is given by the work function ϕ
- Energy from outside the metal that can overcome the work function, for example heat or photons.
- An equivalent potential well model is



Thermionic Emission



- The emission of electrons from the metal at high temperatures “Thermionic Emission” uses the concept of work function to define how much energy is required to remove the electron from the metal to the vacuum.
- We assume all electrons are “free” and mobile in the metal and therefore have kinetic energy

$$E_{kinetic} = \frac{1}{2m}(\rho_x^2 + \rho_y^2 + \rho_z^2), \text{ where } \rho \text{ is the momentum}$$

- In order to escape from the metal, this kinetic energy must be in a direction that allows the electron to escape the metal, assuming the x-direction is the required direction

$$\frac{\rho_x^2}{2m} > \frac{\rho_{x0}^2}{2m} = E_F + \phi$$

Thermionic Emission

- Since there is a probability that the electron will be reflected from the surface discontinuity between the metal and vacuum (using the wave model), we can define a reflection coefficient $r(\rho_x)$ such that the probability of transmission through the boundary is $1-r(\rho_x)$.
- For the number of electrons having momentum between ρ_x and $\rho_x+d\rho_x$

$\frac{\rho_x}{m} N(\rho_x) d\rho_x$, number of electrons arriving at the surface

$(1-r(\rho_x)) \frac{\rho_x}{m} N(\rho_x) d\rho_x$, number of electrons escaping at the surface or transmitting

- We can now define an emission current density (current per cross-sectional area A) for all of the electrons that have momentum in the required direction greater than the barrier momentum ρ_{x0}

$$J = \frac{q}{m} \int_{\rho_{x0}}^{\infty} [1 - r(\rho_x)] \rho_x N(\rho_x) d\rho_x$$

Thermionic Emission

- For a volume with sides dn_x, dn_y, dn_z the number of electrons in the momentum range $[\rho_x, \rho_x + d\rho_x]$; $[\rho_y, \rho_y + d\rho_y]$; $[\rho_z, \rho_z + d\rho_z]$ are given by the product of the density of states and the electron fermi-dirac distribution

Number of states in volume $\rightarrow$ Kinetic energy $\nwarrow$

$$N(\rho_x, \rho_y, \rho_z) d\rho_x d\rho_y d\rho_z = \left(\frac{2}{h^3} \right) \left(1 + \exp \left[\frac{\frac{1}{2m} (\rho_x^2 + \rho_y^2 + \rho_z^2) - E_F}{k_B T} \right] \right)^{-1} d\rho_x d\rho_y d\rho_z$$

- Integrating to get the total number of electrons in the range $[\rho_x, \rho_x + d\rho_x]$

$$N(\rho_x) d\rho_x = \left(\frac{2}{h^3} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{d\rho_y d\rho_z}{1 + \exp \left[\frac{\frac{1}{2m} (\rho_x^2 + \rho_y^2 + \rho_z^2) - E_F}{k_B T} \right]}$$

Thermionic Emission

- Solving the integrals for Gaussians

$$N(\rho_x)d\rho_x = \left(\frac{4\pi mk_B T}{h^3}\right) \exp\left(\frac{E_F}{k_B T}\right) \exp\left(-\frac{\rho_x^2}{2mk_B T}\right) d\rho_x$$

- Then solving for the current density assuming r is independent of momentum direction (an approximation)

$$\begin{aligned} J &= \frac{q}{m} \int_{\rho_{x0}}^{\infty} [1 - r(\rho_x)] \rho_x N(\rho_x) d\rho_x \\ &= \frac{q}{m} \int_{\rho_{x0}}^{\infty} [1 - r] N(\rho_x) d\rho_x = \frac{q}{m} [1 - r] \exp\left(\frac{E_F}{k_B T}\right) \int_{\rho_{x0}}^{\infty} \exp\left(-\frac{\rho_x^2}{2mk_B T}\right) d\rho_x \\ &= \frac{4\pi q m k_B^2}{h^3} (1 - r) T^2 \exp\left(-\frac{\phi}{k_B T}\right) \end{aligned}$$

Thermionic Emission



- Note the dependence on temperature and the value of the work function. A_0 is constant. There is an exponential dependence on temperature.
- In reality, electrons that escape the metal will have several mechanisms to be attracted back (e.g. electrostatic forces from positively charged surface) so we need to replenish the electrons with an external circuit. Putting the metal in a vacuum will also improve emission.